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Technical Notes

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Vibration of Thermally Stressed Composite Cylinders

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I. Introduction

COMPOSITE cylinders represent the skeleton and body of modern high-performance aircraft and naval structures. They are also used in civil engineering structures as storage tanks and in mechanical engineering industries as boilers, water purifying devices, etc. For efficient design, the vibration characteristics of such shell structures is of paramount importance. In many cases, these structures must operate in high or low temperatures that alter the mechanical characteristics of such structures. Therefore, it is important to predict their vibration characteristics when composite cylinders are thermally stressed. This is the purpose of the present study.

Researchers are currently investigating the effect of heating on the natural frequencies of isotropic, orthotropic, and composite plates.[1–4] In general, it is observed that the frequency–temperature curves are nonlinear in nature, greatly affected by the particular lamination. The present study is investigating the effect of the temperature in the fundamental natural frequency of laminated composite shells. Temperature–frequency curves are produced for four different lamination schemes comprising eight layer plies.

II. Computational Model

Figure 1 shows a composite cylinder with all material and geometrical properties. The cylinder has two end diaphragms. It is considered a thin shell structure with a radius-over-thickness ratio equal to 100. Five lamination schemes are considered, namely, 1) $(0/30/-30/90)_s$, 2) $(45/-45/0/90)_s$, 3) $(0/90/0/90)_s$, 4) $(30/-30)_4$, and 5) $(75/-15/0/0)_s$.

The composite cylinder is discretized with a set of triangular shell finite elements[5,6] for a total of 441 nodes, 800 elements, and 2280 degrees of freedom. The fundamental natural frequency is extracted from the eigenvalue problem

$$(K_E + K_G)X = \lambda M X \tag{1}$$

where K_E, K_G, and M are the global elastic stiffness, geometric matrix, and mass matrix, respectively, and λ and X are the eigenvalue (natural frequency) and eigenvector, respectively. The geometric stiffness incorporates the effect of the temperature. A complete geometric nonlinear analysis is conducted in the context that the applied temperature is given in incremental steps. At convergence, the eigenvalue problem (1) is solved, and the fundamental natural frequency is extracted. This process is repeated within the temperature range of interest for selected temperatures and for all of the listed laminations. At the end, appropriate temperature–frequency curves are produced.

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III. Computational Experiments

Figure 2 shows the temperature–frequency curves for a composite cylinder with end diaphragms for an imposed temperature up to $T = 200°C$ for the five different laminations. It is first observed that all curves are highly nonlinear especially for the $(45/-45/0/90)_s$, $(0/30/-30/90)_s$, and $(0/90/0/90)_s$ laminates. Although the quasi-isotropic $(45/-45/0/90)_s$ composite cylinder has the second highest natural frequency at $T = 0°C$ (Fig. 3), it surpasses the frequencies of all other laminations at subsequent temperatures up to $T = 200°C$. It is evident that, for this particular lamination scheme, the composite cylinder shows the highest stiffening effect with increasing temperature, and as such, it presents an excellent design option (Fig. 4). The $(0/30/-30/90)_s$ cylinder has the highest fundamental

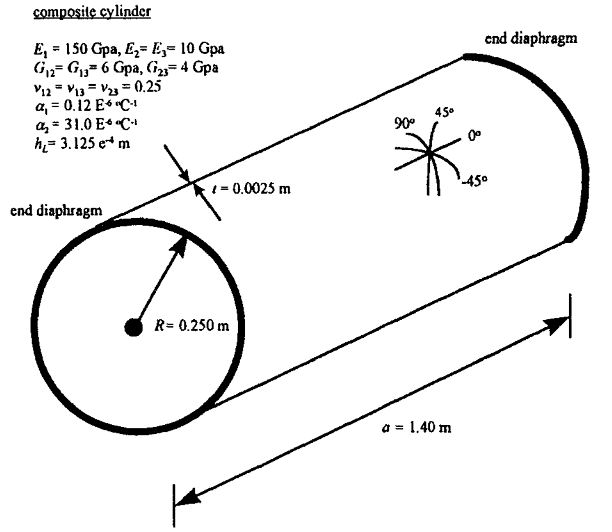

Fig. 1 Geometric and material data for a composite cylinder with end diaphragms.

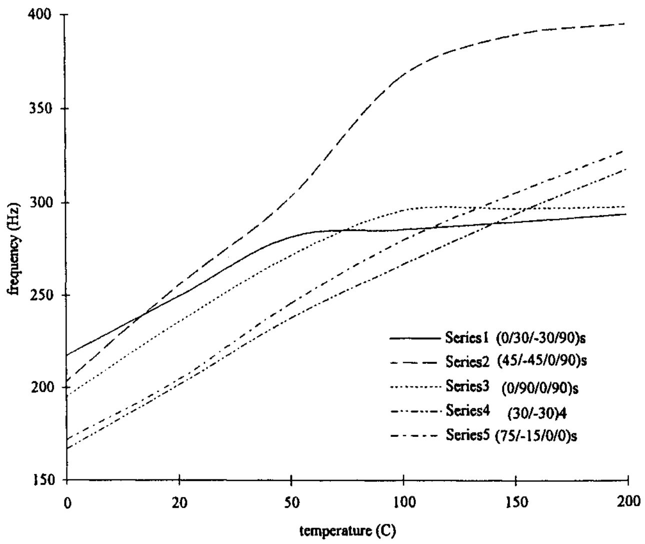

Fig. 2 Frequency–temperature curves for five laminations.

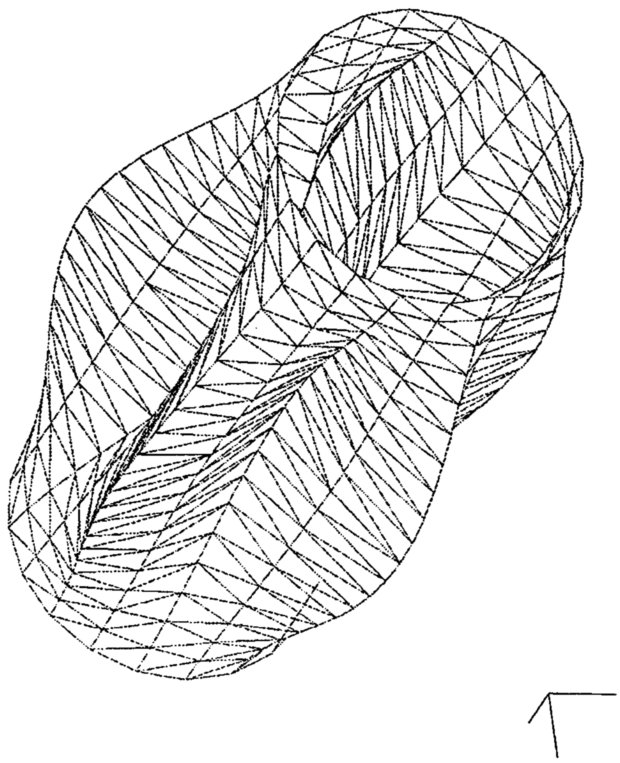

Fig. 3 First vibration mode of the $(45/-45/0/90)_s$ cylinder at $T = 0°C$.

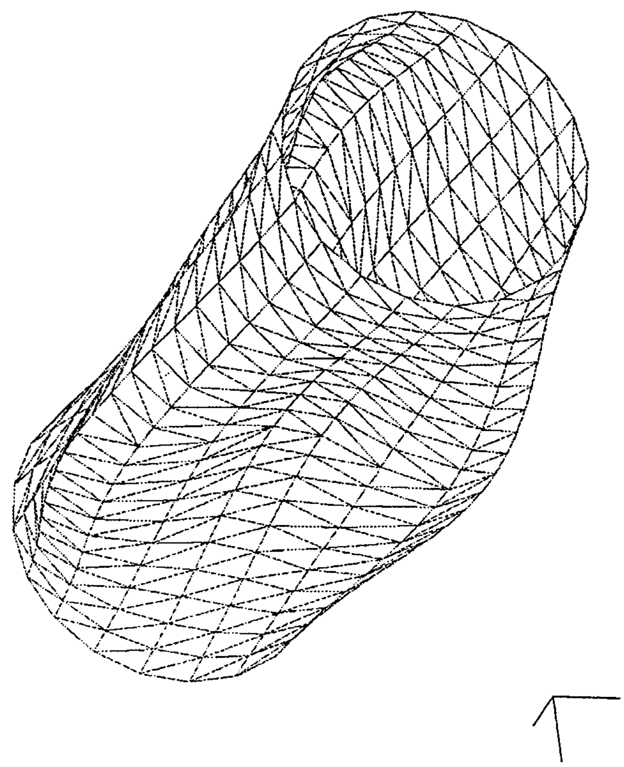

Fig. 4 First vibration mode of the $(45/-45/0/90)_s$ cylinder at $T = 150°C$.

frequency at $T = 0°C$. The latter increases linearly until $T = 50°C$; however, with increasing temperature it remains nearly the same as at $T = 50°C$. The cross-ply $(0/90/0/90)_s$ cylinder displays similar trends after $T = 100°C$. The $(30/-30)_4$, $(75/-15/0/0)_s$ cylindrical shells have the lowest natural frequencies at $T = 0°C$; however, these frequencies increase continuously with temperature and do not display uniform values in some intervals. At $T = 200°C$, they surpass the fundamental frequencies of all laminated cylinders except the $(45/-45/0/90)_s$ composite cylinder.

IV. Conclusions

It is, therefore, concluded that the composite cylinder displays different fundamental frequencies for different laminations when thermally stressed. For some lamination schemes these frequencies may present near values in some temperature intervals. The most interesting observation is that the quasi-isotropic $(45/-45/0/90)_s$ composite cylindrical shell stiffens the most as a result of thermal stressing and represents an excellent design option at elevated environments. Our next study will be concerned with the effect of existing cutouts on the vibration characteristics of stiffened composite shells.

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Stability of Inviscid, Compressible Subsonic Flow in Variable Area Duct

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Nomenclature

A	= amplitude of velocity perturbation
a_0	= speed of sound
$F(x)$	= area of varying cross section of duct
i	= $\sqrt{-1}$
k	= wave number
k_0	= reference wave number
L, λ_0	= characteristic lengths; Eqs. (13) and (14)
M_0	= Mach number, u_0/a_0
t	= time variable
u, ρ, p	= instantaneous velocity, density, and pressure, respectively
u', ρ', p'	= perturbations of velocity, density, and pressure, respectively
u_0, ρ_0, p_0	= steady velocity, density, and pressure, respectively
x	= axial coordinate

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μ, ε = parameters; Eqs. (7) and (12)
ω = circular frequency

Subscript

0 = steady variables

Superscript

' = perturbation variables

I. Introduction

THE purpose of the present investigation is to study the stability of an initial, steady inviscid, compressible subsonic flow in a rigid impermeable duct of varying area. There are several motivations for studying such a problem. For example, one of the unique features of the noise field produced by axial flow fans or compressors is the dominance of spinning modes (or short-wavelength modes) in the fan duct. An additional feature in turbomachinery applications is that the cross section of a fan duct varies along the duct axis. Consequently, to understand the detailed flow behavior, it is imperative to examine, in particular, the stability of the preceding flow subjected to the short-wavelength acoustic perturbations. The analysis is carried out by assuming that the flow is quasi-one-dimensional and that the steady flow variables and the duct cross sections vary slowly over a wavelength of the flow perturbation variables.

II. Basic Equations

The equations of fluid mechanics pertaining to quasi-one-dimensional motion through a duct of varying cross section $F(x)$, in the absence of body forces, are as follows.

Equation of mass:

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} + \rho u\frac{\mathrm{d}\ln F}{\mathrm{d}x} = 0 \tag{1}$$

Equation of motion:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \tag{2}$$

Equation of state:

$$\mathrm{d}p = a_0^2\,\mathrm{d}\rho \tag{3}$$

The initial state of the fluid is assumed to be steady compressible, inviscid subsonic flow. Then the governing equations of the initial steady state, denoted by subscript 0, are as follows.

Equation of mass:

$$\frac{\mathrm{d}(\rho_0 u_0 F)}{\mathrm{d}x} = 0 \tag{4}$$

Equation of motion:

$$u_0\frac{\mathrm{d}u_0}{\mathrm{d}x} + \frac{1}{\rho_0}\frac{\mathrm{d}p_0}{\mathrm{d}x} = 0 \tag{5}$$

Equation of state:

$$\mathrm{d}p_0 = a_0^2\,\mathrm{d}\rho_0 \tag{6}$$

We shall assume that

$$M_0 = u_0/a_0 = \mu \tag{7}$$

where $\mu \ll 1$ and $M_0(x)$ is the local Mach number. Then the equation of mass (4) can be simplified.

Indeed, by using the equation of state (6), the equation of motion (5) can be written in the form

$$\frac{1}{\rho_0}\frac{\mathrm{d}p_0}{\mathrm{d}x} = \frac{1}{\rho_0}\frac{\mathrm{d}p_0}{\mathrm{d}\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}x} = \frac{a_0^2}{\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}x} = -u_0\frac{\mathrm{d}u_0}{\mathrm{d}x} \tag{8}$$

From Eq. (8) one obtains the following relation:

$$\frac{\mathrm{d}\rho_0}{\mathrm{d}x} = -\frac{\rho_0 u_0}{a_0^2}\frac{\mathrm{d}u_0}{\mathrm{d}x} \tag{9}$$

The equation of mass (4) may take the form

$$\frac{\mathrm{d}(\rho_0 u_0 F)}{\mathrm{d}x} = u_0 F\frac{\mathrm{d}\rho_0}{\mathrm{d}x} + \rho_0\frac{\mathrm{d}(u_0 F)}{\mathrm{d}x} = 0 \tag{10}$$

Then after making use of Eqs. (9) and (10) and neglecting the contribution of the term of order, which is equal to μ^2, Eq. (4) becomes

$$\frac{\mathrm{d}(\rho_0 u_0 F)}{\mathrm{d}x} = \rho_0\frac{\mathrm{d}(u_0 F)}{\mathrm{d}x} - F\rho_0\frac{u_0^2}{a_0^2}\frac{\mathrm{d}u_0}{\mathrm{d}x}$$
$$= \rho_0\frac{\mathrm{d}(u_0 F)}{\mathrm{d}x} - F\rho_0\mu^2\frac{\mathrm{d}u_0}{\mathrm{d}x} \approx \rho_0\frac{\mathrm{d}(u_0 F)}{\mathrm{d}x} = 0 \tag{11}$$

We shall further assume that

$$\varepsilon = \lambda_0/L = \mu^{\frac{1}{2}} \tag{12}$$

where L, λ_0 are the characteristic lengths. A length L characterizes the variation with x of the steady variables u_0 and F, defined as follows:

$$\frac{1}{L} = \max\left\{\frac{\mathrm{d}\ln u_0}{\mathrm{d}x}, \frac{\mathrm{d}\ln F}{\mathrm{d}x}\right\} \tag{13}$$

in the region of interest, and λ_0 is the wavelength

$$\lambda_0 = 2\pi/k_0 \tag{14}$$

where $k = \omega/a_0$ is the reference wave number associated with a wave of frequency ω propagating in a homogeneous medium at rest with speed of sound a_0. Note that the assumption (12) means that the variations of $u_0(x)$, $M_0(x)$ are slow over a wavelength λ_0.

The perturbation variables u', ρ', p' are such that

$$\rho' \ll \rho_0, \qquad p' \ll p_0, \qquad u' \ll u_0 \tag{15}$$

After eliminating the terms of the order, which is equal to or higher than μ^2, one obtains the following equations for the perturbation variables.

Equation of mass:

$$\frac{\partial \rho'}{\partial t} + u_0\frac{\partial \rho'}{\partial x} + \rho_0 u'\frac{\mathrm{d}\ln F}{\mathrm{d}x} + \rho_0\frac{\partial u'}{\partial x} = 0 \tag{16}$$

Equation of motion:

$$\frac{\partial u'}{\partial t} + \frac{\partial(u_0 u')}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial x} = 0 \tag{17}$$

Equation of state:

$$p' = a_0^2\rho' \tag{18}$$

It is important to notice that the equations obtained do not contain the derivative of the function $\rho_0(x)$ with respect to the x variable because the terms involving $\mathrm{d}\rho_0/\mathrm{d}x$ are of the order that is equal to or higher than μ^2.

III. Wave Equation in a Nonuniform Duct

From Eqs. (16–18), one derives through appropriate differentiations with respect to time and distance and elimination of the terms of the order, which is equal to or higher than μ^2, a second-order partial differential equation in the perturbation velocity u':

$$\frac{\partial^2 u'}{\partial t^2} + u_0 \frac{\partial^2 u'}{\partial x \partial t} + \frac{\mathrm{d}u_0}{\mathrm{d}x}\frac{\partial u'}{\partial t} - a_0^2 \frac{\mathrm{d}\ln F}{\mathrm{d}x}\frac{\partial u'}{\partial x} - a_0^2 \frac{\mathrm{d}^2 \ln F}{\mathrm{d}x^2} u' - a_0^2 \frac{\partial^2 u'}{\partial x^2} = 0 \tag{19}$$

The assumptions involved in the derivation of Eq. (19) are

$$\frac{\partial^2 u'}{\partial t^2} \approx u_0 \omega^2 \tag{20}$$

$$u_0 \frac{\partial^2 u'}{\partial x \partial t} \approx \frac{u_0^2 \omega}{\lambda_0} \tag{21}$$

$$\frac{\mathrm{d}u_0}{\mathrm{d}x}\frac{\partial u'}{\partial t} \approx \frac{u_0^2 \omega}{L} \tag{22}$$

$$a_0^2 \frac{\mathrm{d}\ln F}{\mathrm{d}x}\frac{\partial u'}{\partial x} \approx \frac{a_0^2 u_0}{\lambda_0 L} \tag{23}$$

$$a_0^2 \frac{\mathrm{d}^2 \ln F}{\mathrm{d}x^2} u' \approx \frac{a_0^2 u_0}{L^2} \tag{24}$$

$$a_0^2 \frac{\partial^2 u'}{\partial x^2} \approx \frac{a_0^2 u_0}{\lambda_0^2} \tag{25}$$

$$\frac{\partial}{\partial x}\left(u_0 \frac{\partial \rho'}{\partial x}\right) = \frac{\mathrm{d}u_0}{\mathrm{d}x}\frac{\partial p'}{\partial x}\frac{1}{a_0^2} + u_0 \frac{\partial^2 p'}{\partial x^2}\frac{1}{a_0^2} \approx \frac{u_0^3 \rho_0}{L \lambda_0 a_0^2} + \frac{\rho_0 u_0^3}{\lambda_0^2 a_0^2} \tag{26}$$

$$\frac{1}{\rho_0^2}\frac{\mathrm{d}\rho_0}{\mathrm{d}x}\frac{\partial p}{\partial t} = -\frac{u_0}{a_0^2 \rho_0}\frac{\mathrm{d}u_0}{\mathrm{d}x}\frac{\partial p}{\partial t} \approx \frac{u_0^4 \omega}{a_0^2 L} \tag{27}$$

$$-\frac{u_0}{\rho_0^2}\frac{\mathrm{d}\rho_0}{\mathrm{d}x}\frac{\partial p}{\partial x} = \frac{u_0^2}{a_0^2 \rho_0}\frac{\mathrm{d}u_0}{\mathrm{d}x}\frac{\partial p}{\partial x} \approx \frac{u_0^5}{a_0^2 L \lambda} \tag{28}$$

Analogously with Eqs. (16) and (17), Eq. (19) does not contain the derivative of the function $\rho_0(x)$ with respect to the x variable, too.

IV. Solution

For the stability analysis of the base compressible, inviscid subsonic flow in a duct of varying area, the approach of Chomaz et al.[1] and Huerre and Monkewitz[2] will be used. As in the approach,[1,2] the slow variable $X = \varepsilon x$ is introduced, and the functions F, u_0 are assumed to depend on the parameter X; therefore,

$$F = F(X), \qquad u_0 = u_0(X)$$

The solution to Eq. (19) is sought in the following form[1,2]:

$$u' = \Phi(x, X) e^{i\omega t}$$

where $\Phi(x, X)$ is the eigenfunction, which in a general case is to be determined by the boundary conditions, and ω is the fundamental frequency. Following the approach[1,2] the local dispersion relationship corresponding to Eq. (19) has been obtained by freezing the slow variable X in Eq. (19) and supposing

$$\Phi(x, X) = A e^{-ikx}$$

where A is the amplitude of u'. This local dispersion relationship can be written as follows:

$$\omega = \Omega_{1,2}(k, X) \equiv \frac{1}{2}\left[u_0(X)k - i u_0(X)\varepsilon \frac{\mathrm{d}\ln F}{\mathrm{d}X}\right] \pm \sqrt{\left\{\frac{1}{4}\left[u_0(X)k + i\varepsilon \frac{\mathrm{d}u_0}{\mathrm{d}X}\right]^2 + (ka_0)^2 - a_0^2 \varepsilon^2 \frac{\mathrm{d}^2 \ln F}{\mathrm{d}X^2} + ika_0^2 \varepsilon \frac{\mathrm{d}\ln F}{\mathrm{d}X}\right\}} \tag{29}$$

where k is the wave number associated with the wave of frequency ω propagating in the duct. In Eq. (29) the distance X and the wave number k should be regarded as parameters.

Dispersion relationship (29) can be greatly simplified. By making use of Eqs. (7), (12), and (20–28) and neglecting the contribution of terms of order, which is equal to or higher than μ^2, the square root term of Eq. (29) can be written as

$$\begin{aligned}
&\sqrt{\left\{\frac{1}{4}\left[u_0(X)k + i\varepsilon \frac{\mathrm{d}u_0}{\mathrm{d}X}\right]^2 + (ka_0)^2 - a_0^2 \varepsilon^2 \frac{\mathrm{d}^2 \ln F}{\mathrm{d}X^2} + ika_0^2 \varepsilon \frac{\mathrm{d}\ln F}{\mathrm{d}X}\right\}} \\
&= ka_0 \sqrt{\left\{\frac{1}{4}\left[\frac{u_0(X)}{a_0}\right]^2 - \frac{1}{4}\left(\frac{\varepsilon}{ka_0}\frac{\mathrm{d}u_0}{\mathrm{d}X}\right)^2 + \frac{\varepsilon}{2} i \frac{u_0(X)}{ka_0^2}\frac{\mathrm{d}u_0}{\mathrm{d}X} + 1 - \frac{\varepsilon^2}{k^2}\frac{\mathrm{d}^2 \ln F}{\mathrm{d}X^2} + i\frac{\varepsilon}{k}\frac{\mathrm{d}\ln F}{\mathrm{d}X}\right\}} \approx ka_0 \sqrt{\left(1 - \frac{\varepsilon^2}{k^2}\frac{\mathrm{d}^2 \ln F}{\mathrm{d}X^2} + i\frac{\varepsilon}{k}\frac{\mathrm{d}\ln F}{\mathrm{d}X}\right)} \\
&= \left(\pm \frac{a_0}{2}\varepsilon \frac{\mathrm{d}\ln F}{\mathrm{d}X}\right) \Bigg/ \sqrt{\left\{\frac{1}{2}\sqrt{\left[1 - \frac{2}{k^2}\varepsilon^2 \frac{\mathrm{d}^2 \ln F}{\mathrm{d}X^2} + \left(\frac{\varepsilon^2}{k^2}\frac{\mathrm{d}^2 \ln F}{\mathrm{d}X^2}\right)^2 + \left(\frac{\varepsilon}{k}\frac{\mathrm{d}\ln F}{\mathrm{d}X}\right)^2\right]} - \frac{1}{2} + \frac{\varepsilon^2}{2k^2}\frac{\mathrm{d}^2 F}{\mathrm{d}X^2}\right\}} \\
&\pm ika_0 \sqrt{\left[\frac{1}{2}\sqrt{1 - \frac{2}{k^2}\varepsilon^2 \frac{\mathrm{d}^2 \ln F}{\mathrm{d}X^2} + \left(\frac{\varepsilon^2}{k^2}\frac{\mathrm{d}^2 \ln F}{\mathrm{d}X^2}\right)^2 + \left(\frac{\varepsilon}{k}\frac{\mathrm{d}\ln F}{\mathrm{d}X}\right)^2} - \frac{1}{2} + \frac{\varepsilon^2}{2k^2}\frac{\mathrm{d}^2 F}{\mathrm{d}X^2}\right]} \approx ka_0 \left(\pm 1 \pm i\frac{\varepsilon}{2k}\frac{\mathrm{d}\ln F}{\mathrm{d}X}\right)
\end{aligned} \tag{30}$$

On inserting Eq. (30) into Eq. (29), we obtain

$$\frac{\omega}{a_0} = \frac{\Omega_{1,2}(k, X)}{a_0} \equiv k\left[\frac{1}{2}M_0(X) \pm 1\right] - i[M_0(X) \mp 1]\frac{\varepsilon}{2}\frac{\mathrm{d}\ln F}{\mathrm{d}X} \tag{31}$$

V. Results

As can be seen in Eq. (31), ω/a_0 contains the real $k[0.5M_0(X) \pm 1]$ and imaginary $-0.5[M_0(X) \mp 1]\varepsilon(\mathrm{d}\ln F/\mathrm{d}X)$ parts. Therefore, by definition[3,4] the flow in the duct is unstable because, according to Eq. (31), for all real k there exists a negative imaginary part of ω/a_0, for example, at the points X for which the condition $0.5[M_0(X) \mp 1]\,\varepsilon(\mathrm{d}\ln F/\mathrm{d}X) > 0$ is met. Obviously, at the points X for which $\mathrm{d}\ln F/\mathrm{d}X = 0$, the flow is stable for all real k. Note that $[0.5M_0(X) \pm 1]$ is nothing but the ratio of the speed of propagation of the wave, $0.5u_0(X) \pm a_0$, to the speed of sound a_0 at the point under consideration.

VI. Conclusions

A satisfactory physical explanation of this phenomenon is not easily given. So, the instability is probably caused by the unlimited acoustic energy influx associated with conditions that, in turn, are directly related to the assumptions involved in the derivation of Eq. (19).

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